12-5 Adding Probabilities

Main Ideas

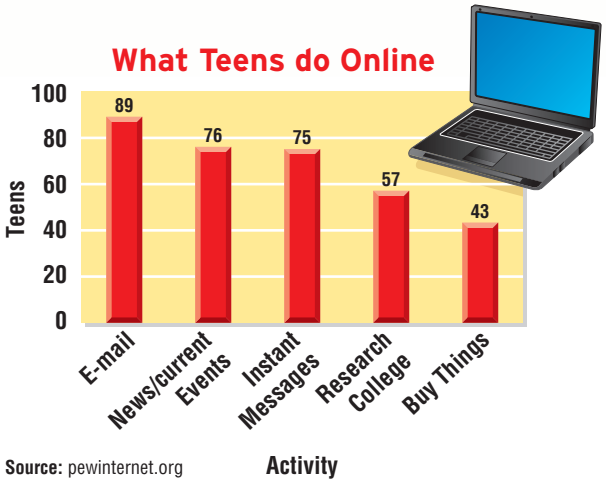
- Find the probability of mutually exclusive events.
- Find the probability of inclusive events.

New Vocabulary

simple event
compound event
mutually exclusive events
inclusive events

GET READY for the Lesson

The graph shows the results of a survey about what teens do online. Determining the probability that a randomly selected teen sends/reads e-mail or buys things online requires adding probabilities.



Mutually Exclusive Events When you roll a die, an event such as rolling a 1 is called a **simple event** because it cannot be broken down into smaller events. An event that consists of two or more simple events is called a **compound event**. For example, the event of rolling an odd number or a number greater than 5 is a compound event because it consists of the simple events rolling a 1, rolling a 3, rolling a 5, or rolling a 6.

When there are two events, it is important to understand how they are related before finding the probability of one or the other event occurring. Suppose you draw a card from a standard deck of cards. What is the probability of drawing a 2 or an ace? Since a card cannot be both a 2 *and* an ace, these are called **mutually exclusive events**. That is, the two events cannot occur at the same time. The probability of drawing a 2 or an ace is found by adding their individual probabilities.

$$P(2 \text{ or ace}) = P(2) + P(\text{ace}) \quad \text{Add probabilities.}$$

$$= \frac{4}{52} + \frac{4}{52} \quad \text{There are 4 twos and 4 aces in a deck.}$$

$$= \frac{8}{52} \text{ or } \frac{2}{13} \quad \text{Simplify.}$$

The probability of drawing a 2 or an ace is $\frac{2}{13}$.

Study Tip

Formula
This formula can be extended to any number of mutually exclusive events.

KEY CONCEPT *Probability of Mutually Exclusive Events*

Words If two events, A and B, are mutually exclusive, then the probability that A or B occurs is the sum of their probabilities.

Symbols $P(A \text{ or } B) = P(A) + P(B)$

EXAMPLE 1 Two Mutually Exclusive Events

Keisha has a stack of 8 baseball cards, 5 basketball cards, and 6 soccer cards. If she selects a card at random from the stack, what is the probability that it is a baseball or a soccer card?

These are mutually exclusive events, since the card cannot be both a baseball card *and* a soccer card. Note that there is a total of 19 cards.

$P(\text{baseball or soccer}) = P(\text{baseball}) + P(\text{soccer})$ Mutually exclusive events

$= \frac{8}{19} + \frac{6}{19}$ or $\frac{14}{19}$ Substitute and add.

The probability that Keisha selects a baseball or a soccer card is $\frac{14}{19}$.

CHECK Your Progress

1. One teacher must be chosen to supervise a senior class fund-raiser. There are 12 math teachers, 9 language arts teachers, 8 social studies teachers, and 10 science teachers. If the teacher is chosen at random, what is the probability that the teacher is either a language arts teacher or a social studies teacher?

To extend the formula to more than two events, add the probabilities for all of the events.

EXAMPLE 2 Three Mutually Exclusive Events

There are 7 girls and 6 boys on the junior class homecoming committee. A subcommittee of 4 people is being chosen at random to decide the theme for the class float. What is the probability that the subcommittee will have at least 2 girls?

At least 2 girls means that the subcommittee may have 2, 3, or 4 girls. It is not possible to select a group of 2 girls, a group of 3 girls, and a group of 4 girls all in the same 4-member subcommittee, so the events are mutually exclusive. Add the probabilities of each type of committee.

$P(\text{at least 2 girls}) = P(2 \text{ girls}) + P(3 \text{ girls}) + P(4 \text{ girls})$

$= \frac{C(7, 2) \cdot C(6, 2)}{C(13, 4)} + \frac{C(7, 3) \cdot C(6, 1)}{C(13, 4)} + \frac{C(7, 4) \cdot C(6, 0)}{C(13, 4)}$ (2 girls, 2 boys; 3 girls, 1 boy; 4 girls, 0 boys)

$= \frac{315}{715} + \frac{210}{715} + \frac{35}{715}$ or $\frac{112}{143}$ Simplify.

The probability of at least 2 girls on the subcommittee is $\frac{112}{143}$ or about 0.78.

Study Tip

Choosing a Committee

$C(13, 4)$ refers to choosing 4 subcommittee members from 13 committee members. Since order does not matter, the number of combinations is found.

CHECK Your Progress

2. The Cougar basketball team can send 5 players to a basketball clinic. Six guards and 5 forwards would like to attend the clinic. If the players are selected at random, what is the probability that at least 3 of the players selected to attend the clinic will be forwards?

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Inclusive Events What is the probability of drawing a king or a spade from a standard deck of cards? Since it is possible to draw a card that is both a king and a spade, these events are not mutually exclusive. These are called **inclusive events**.

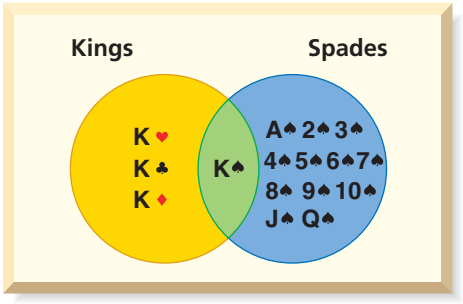
P(king)	P(spade)	P(spade, king)
$\frac{4}{52}$	$\frac{13}{52}$	$\frac{1}{52}$
1 king in each suit	spades	king of spades

Study Tip

Common Misconception

In mathematics, unlike everyday language, the expression *A* or *B* allows the possibility of both *A* and *B* occurring.

In the first two fractions above, the probability of drawing the king of spades is counted twice, once for a king and once for a spade. To find the correct probability, you must subtract P(king of spades) from the sum of the first two probabilities.



$$P(\text{king or spade}) = P(\text{king}) + P(\text{spade}) - P(\text{king of spades})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \text{ or } \frac{4}{13}$$

The probability of drawing a king or a spade is $\frac{4}{13}$.

KEY CONCEPT *Probability of Inclusive Events*

Words If two events, *A* and *B*, are inclusive, then the probability that *A* or *B* occurs is the sum of their probabilities decreased by the probability of both occurring.

Symbols $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

EXAMPLE **Inclusive Events**

3 **EDUCATION** **Suppose that of 1400 students, 550 take Spanish, 700 take biology, and 400 take both Spanish and biology. What is the probability that a student selected at random takes Spanish or biology?**

Since some students take both Spanish and biology, the events are inclusive.

$$P(\text{Spanish}) = \frac{550}{1400} \qquad P(\text{biology}) = \frac{700}{1400} \qquad P(\text{Spanish and biology}) = \frac{400}{1400}$$

$$P(\text{Spanish or biology}) = P(\text{Spanish}) + P(\text{biology}) - P(\text{Spanish and biology})$$

$$= \frac{550}{1400} + \frac{700}{1400} - \frac{400}{1400} \text{ or } \frac{17}{28} \quad \text{Substitute and simplify.}$$

The probability that a student selected at random takes Spanish or biology is $\frac{17}{28}$.

CHECK Your Progress

3. Sixty plastic discs, each with one of the numbers from 1 to 60, are in a bag. LaTanya will win a game if she can pull out any disc with a number divisible by 2 or 3. What is the probability that LaTanya will win?

Concepts in Motion
Interactive Lab
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CHECK Your Understanding

Examples 1–3 (pp. 711–712)

A die is rolled. Find each probability.

1. P(1 or 6)
2. P(at least 5)
3. P(less than 3)
4. P(even or prime)
5. P(multiple of 3 or 4)
6. P(multiple of 2 or 3)

Examples 2, 3 (pp. 711–712)

A card is drawn from a standard deck of cards. Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

7. P(6 or king)
8. P(queen or spade)

Example 2 (p. 711)

9. **SCHOOL** There are 8 girls and 8 boys on the Student Senate. Three of the students are seniors. What is the probability that a person selected from the Student Senate is not a senior?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
10–19	1, 2
20–23	1–3
24–29	3

Jesse has eight friends who have volunteered to help him with a school fundraiser. Five are boys and 3 are girls. If he randomly selects 3 friends to help him, find each probability.

10. P(2 boys or 2 girls)
11. P(all boys or all girls)
12. P(at least 2 girls)
13. P(at least 1 boy)

Six girls and eight boys walk into a video store at the same time. There are six salespeople available to help them. Find the probability that the salespeople will first help the given numbers of girls and boys.

14. P(4 girls, 2 boys or 4 boys, 2 girls)
15. P(5 girls, 1 boy or 5 boys, 1 girl)
16. P(all girls or all boys)
17. P(at least 4 boys)
18. P(at least 5 girls or at least 5 boys)
19. P(at least 3 girls)

For Exercises 20–23, determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

20. There are 4 algebra books, 3 literature books, and 2 biology books on a shelf. If a book is randomly selected, what is the probability of selecting a literature book or an algebra book?
21. A die is rolled. What is the probability of rolling a 5 or a number greater than 3?
22. In the Math Club, 7 of the 20 girls are seniors, and 4 of the 14 boys are seniors. What is the probability of randomly selecting a boy or a senior to represent the Math Club at a statewide math contest?
23. A card is drawn from a standard deck of cards. What is the probability of drawing an ace or a face card? (*Hint:* A face card is a jack, queen, or king.)
24. One tile with each letter of the alphabet is placed in a bag, and one is drawn at random. What is the probability of selecting a vowel or a letter from the word *function?*
25. Each of the numbers from 1 to 30 is written on a card and placed in a bag. If one card is drawn at random, what is the probability that the number is a multiple of 2 or a multiple of 3?

Two cards are drawn from a standard deck of cards. Find each probability.

26. P(both queens or both red)
27. P(both jacks or both face cards)
28. P(both face cards or both black)
29. P(both either black or an ace)

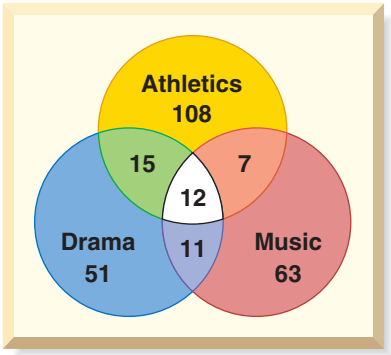
GAMES **For Exercises 30–35, use the following information.**
A certain game has two stacks of 30 tiles with pictures on them. In the first stack of tiles, there are 10 dogs, 4 cats, 5 balls, and 11 horses. In the second stack of tiles, there are 3 flowers, 8 fish, 12 balls, 2 cats, and 5 horses. The top tile in each stack is chosen. Find each probability.

30. P(each is a ball)
31. P(neither is a horse)
32. P(exactly one is a ball)
33. P(exactly one is a fish)
34. P(both are a fish)
35. P(one is a dog and one is a flower)

BASEBALL **For Exercises 36–38, use the following information.**
Albert and Paul are on the school baseball team. Albert has a batting average of .4, and Paul has a batting average of .3. That means that Albert gets a hit 40% of his at bats and Paul gets a hit 30% of his times at bat. What is the probability that—

36. both Albert and Paul are able to get hits their first time at bat?
37. neither Albert nor Paul is able to get a hit their first time at bat?
38. at least one of the two friends is able to get a hit their first time at bat?

SCHOOL **For Exercises 39–41, use the Venn diagram that shows the number of participants in extracurricular activities for a junior class of 324 students.**
Determine each probability if a student is selected at random from the class.



39. P(drama or music)
40. P(drama or athletics)
41. P(athletics and drama, or music and athletics)

EXTRA PRACTICE
See pages 918, 937.
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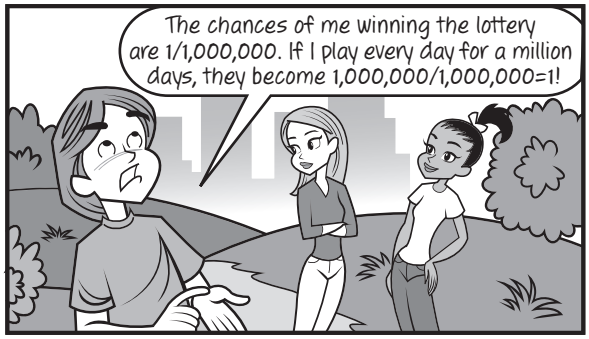
H.O.T. Problems

42. **REASONING** What is wrong with the conclusion in the comic?

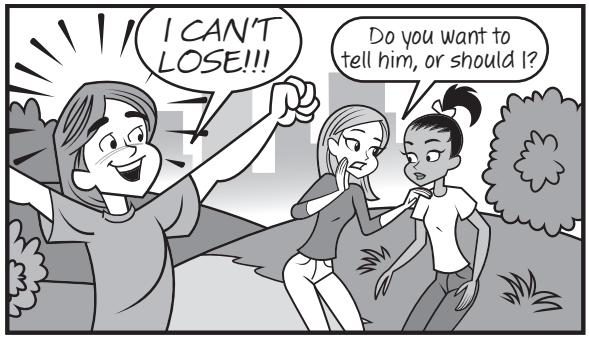


43. **OPEN ENDED** Describe two mutually exclusive events and two inclusive events.

44. **CHALLENGE** A textbook gives the following probability equation for events A and B that are mutually exclusive or inclusive.

$P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$

Is this correct? Explain.



45. ***Writing in Math*** Use the information on page 710 to explain how probability applies to what teens do online. Include an explanation of whether the events listed in the graphic are mutually exclusive or inclusive.

STANDARDIZED TEST PRACTICE

46. ACT/SAT In a jar of red and white gumballs, the ratio of white gumballs to red gumballs is 5:4. If the jar contains a total of 180 gumballs, how many of them are red?

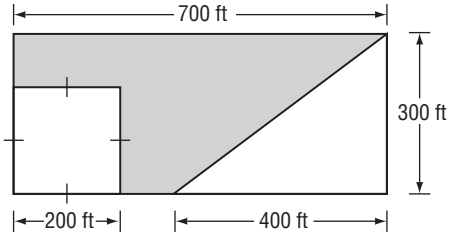
A 45

B 64

C 80

D 100

47. REVIEW What is the area of the shaded part of the rectangle below?



F 90,000 ft^2

G 110,000 ft^2

H 130,000 ft^2

J 150,000 ft^2

Spiral Review

A die is rolled three times. Find each probability. (Lesson 12-4)

48. P(1, then 2, then 3)

49. P(no 4s)

50. P(three 1s)

51. P(three even numbers)

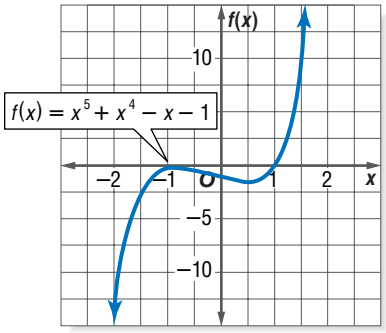
52. BOOKS Dan has twelve books on his shelf that he has not read yet. There are seven novels and five biographies. He wants to take four books with him on vacation. What is the probability that he randomly selects two novels and two biographies? (Lesson 12-3)

Find the sum of each series. (Lessons 11-2 and 11-4)

53. $2 + 4 + 8 + \cdots + 128$

54. $\sum_{n=1}^{3} (5n - 2)$

55. Use the graph of the polynomial function at the right to determine at least one binomial factor of the polynomial. Then find all factors of the polynomial. (Lesson 6-7)



SPEED SKATING **For Exercises 56 and 57, use the following information.**
In 2001, Catriona LeMay Doan set a world record for women's speed skating by skating approximately 13.43 meters per second in the 500-meter race. (Lesson 2-6)

56. Suppose she could maintain that speed. Write an equation that represents how far she could travel in t seconds.

57. What type of function does the equation in Exercise 56 represent?

GET READY for the Next Lesson

PREREQUISITE SKILL **Find the mean, median, mode, and range for each set of data. Round to the nearest hundredth, if necessary.** (Pages 759 and 760)

58. 298, 256, 399, 388, 276

59. 3, 75, 58, 7, 34

60. 4.8, 5.7, 2.1, 2.1, 4.8, 2.1

61. 80, 50, 65, 55, 70, 65, 75, 50

62. 61, 89, 93, 102, 45, 89

63. 13.3, 15.4, 12.5, 10.7